Reforming the mathematics core for engineers and everyone else

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Like many undergraduate departments in the mathematical sciences, the Department of Mathematics at Trinity University plays dual roles. We are committed to educating our mathematics majors for careers in industry, academics, the actuarial profession, and teaching. Yet the preponderance of our instruction is in the mathematical training of students in other disciplines, particularly the other science, technology, engineering, and mathematics (STEM) disciplines. In institutions with engineering programs mathematicians invest much time and effort teaching mathematics to engineering students. At Trinity, we often hear from our colleagues in the Department of Engineering Science about the perceived needs of their majors - usually the students' weaknesses in specific mathematical techniques or problem-solving skills. A particular instance arose when the engineers suggested that we revisit the six-semester course of mathematics required of their majors. The problem, in a nutshell, was to address weaknesses they identified in students completing the sequence, primarily in using calculus to solve applied problems and in applying probability and statistics to empirical investigations.

We wanted to respond to the needs of Engineering Science, but given the size and mission of our institution, we could not afford to create a separate track of instruction in calculus, linear algebra, and differential equations just for engineering students. That said, having just completed a revision of the requirements for a B.A. in Mathematics, we did not want a new core curriculum in the first two years that would deviate from our goals for our own majors. Thus we were faced with the following challenge: To modify core mathematics instruction to improve its relevance for students in other STEM disciplines (in our case, engineering) without sacrificing the content necessary to prepare mathematics majors for a rigorous B.A. program.

The context for reforming the mathematics sequence for engineering students

Trinity University is a mostly undergraduate institution of less than 3000 students that emphasizes liberal arts and sciences, as well as a few select professional programs. It has strong programs in the natural sciences that emphasize undergraduate research. Trinity's B.S. in Engineering Science is a single degree program encompassing the spectrum of engineering disciplines. The engineering curriculum places a heavy emphasis upon two pedagogical strategies: engineering design and fundamental training in mathematics and the physical sciences. In particular, the engineering students must complete a six-semester sequence of mathematics, resulting in a minor in mathematics. The department has nine full-time faculty and matriculates 25-35 majors each year. The Department of
Mathematics at Trinity has nine full-time faculty in mathematics, applied mathematics, and statistics. In addition to teaching courses for the major, they are responsible for courses to satisfy institutional graduation requirements ("understandings") and core training to STEM disciplines (primarily in the calculus sequence). Engineering science represents a plurality of students enrolled in lower division mathematics courses.

In the fall of 2000, the chair of Engineering Science approached the mathematicians with a problem. The latest Accreditation Board for Engineering and Technology (ABET) report indicated that the engineering students demonstrated weaknesses in probability and statistics. To address this the chair asked us to consider creating specific coursework in probability and statistics. This curricular issue arose contemporaneously with another event. We had just sent a cooperative team of two mathematicians and one engineer to an NSF/COMAP INTERMATH Reforming the Core Workshop at the U.S. Military Academy. Attending this meeting as a team gave us ample time (such as while waiting in the airport) to interact. What came to light was that, in the eyes of our faculty colleagues in engineering, not just probability and statistics but the entire six-semester sequence of mathematics courses for engineers was in need of an overhaul.

**The reform process - an interdepartmental effort**

The first step after identifying the primary problem was to hold an interdepartmental retreat. We held a weekend retreat in the spring of 2001 at a ranch west of San Antonio and scheduled ample time for socializing, as well as serious discussion. At the retreat the engineers expressed their needs, and the mathematicians countered with their concerns about how potential changes would impact the mathematics students. As a result we identified the following issues:

* For reasons stated earlier, creating a separate track for engineers was not a desirable option.
* Despite that, we still needed to fit a separate probability and statistics for engineers into the curriculum. Our current courses were not sufficient to satisfy the needs of the engineering students. In sort of a "Goldilocks" scenario, the first-year statistics course was too little but the full-year third-year sequence was too much. The engineers needed one that was "just right" and relevant to their emphasis in design.
* We needed to obey the law of "conservation of coursework" for the engineering students. To maintain a six-course load, something had to go. The engineers suggested dropping linear algebra but we saw numerous negative consequences of that outcome. Most obvious is the question of where the engineering students, particularly those in the electrical engineering emphasis, would learn the linear algebra necessary for ordinary and partial differential equations (such as linear systems of differential equations and Sturm-Liouville problems).
* The topics in the calculus sequence were not timed well with instruction in engineering and the physical sciences. Transcendental functions, elementary differential equations and vector algebra were all needed earlier.
* We desired to minimize the impact on other constituent departments that rely on
these courses for core mathematics instruction.

The next step was to form three interdisciplinary task forces to address the problems and return recommendations to the departments. We arranged the task forces by topic: the calculus sequence, ordinary and partial differential equations, and probability and statistics. Time constraints during the academic year forced the task forces to work primarily over the summer of 2002, but that actually gave the respective task forces more uninterrupted time to collaborate on areas of overlap, such as calculus and differential equations. The task forces’ reports addressed three basic areas: recommendations on content revision, identification of instructor resources (including textbook recommendations), and plans for assessment. Their reports were collected and assimilated, and the two departments reconvened to discuss them.

Outcomes

Discussing the task force reports led us to a revised plan for the six-semester sequence. The old and new mathematics sequence for the engineering science students are listed below in chronological order.

<table>
<thead>
<tr>
<th>Old sequence:</th>
<th>New sequence:</th>
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<tbody>
<tr>
<td>1. Calculus I</td>
<td>Calculus I*</td>
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<tr>
<td>2. Calculus II</td>
<td>Calculus II*</td>
</tr>
<tr>
<td>3. Applied Linear Algebra</td>
<td>Vector Calculus*</td>
</tr>
<tr>
<td>4. Differential Equations</td>
<td>Differential Equations*</td>
</tr>
<tr>
<td>5. Vector Calculus</td>
<td>Engineering Probability and Statistics**</td>
</tr>
<tr>
<td>6. Advanced Calculus for Application</td>
<td>Partial Differential Equations**</td>
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(* indicates a modified course, ** indicates a new course)

A few courses received major overhauls in content. Calculus I now has early transcendentals, Calculus II has early differential equations, and Vector Calculus ends with multivariate integration (a topic formerly postponed until Advanced Calculus for Application in the sixth semester). In addition, two new courses appear. A new course, Partial Differential Equations, replaces Advanced Calculus for Application, covering Sturm-Liouville problems and other boundary value problems, Fourier analysis, and separation of variables techniques. The new Engineering Probability and Statistics course covers multivariate regression, ANOVA, and testing and reliability. In order to remove Applied Linear Algebra from the sequence, we inserted “just-in-time” instruction in basic linear algebra in both Differential Equations and Partial Differential Equations.

We established procedures for assessment. The task forces remain in place to perform annual monitoring not only of content match, but also of student performance in subsequent mathematics courses and engineering courses. The data they will examine will be quantitative measures of student performance outcomes, as well as interviews.
with both students and instructors. Task forces have the responsibility to identify problems and to recommend necessary adjustments. In addition we plan to track attrition rate in both majors to ensure that this is not increased by the more compact curriculum.

After both departments unanimously approved the proposal, we created a timetable for conversion to the new curriculum and then took our proposal to the other STEM departments. (We also included the Department of Economics, due to the large number of students in the mathematical/theoretical economics emphasis). To our delight, each department wholeheartedly and enthusiastically endorsed the changes, recognizing that the proposed changes also addressed their students' needs. Thus, when we went to the University Curriculum Council for approval in the fall of 2002, we had the blessing of numerous constituencies. The plan was approved, and implementation began in the fall of 2003.

The overall outcome of our revision efforts was to create a core curriculum for engineers that features early transcendental functions and an introduction to elementary first- and second-order differential equations in single-variable calculus, greater emphasis on vector analysis and vector integration in vector calculus, basic linear algebra in differential equations, and a more rigorous yet more applicable course in partial differential equations. The new curriculum has the advantage of providing timely instruction in topics that are the primary mathematical tools for models and analysis in STEM disciplines.

However, we realized that the alterations to the timing and coverage of content will also benefit mathematics majors. They will see a more lively and more relevant core in their first two years, with no loss in rigor. In addition they will continue to take linear algebra, a crucial course to prepare them for the more theoretical upper-division mathematics courses. Our initial reluctance to alter these crucial preparatory courses for our majors was quickly displaced with enthusiasm when a departmental team attended an MER Forum Workshop at Washington University in May 2002. Seeing what other mathematics departments had done to broaden the scope of their curricula and talking to our peers about ways to draw more students into the mathematics major gave us the courage to go forward with our curricular changes.

**Solving a few problems "on the fly"**

In order to conserve the number of sections of mathematics courses taught, the new probability and statistics course would replace one section of our introductory statistics, and one section would be sufficient for a whole cohort of engineering students.

In fact, we may realize an economy in the number of sections taught by removing the linear algebra requirement. Linear algebra is still required for math majors, but is now an elective for engineers (and is especially recommended for the electrical engineering emphasis), but we anticipate that the overall enrollments would decline by a section a
year. A major issue with dropping Applied Linear Algebra from the sequence was that the course is required for the minor in mathematics. After much heated debate, the mathematicians agreed to create a new minor in applied mathematics that the six-course sequence for engineers would satisfy.

The compression of the calculus sequence to accommodate early transcendental functions and differential equations raised the question of preparation. Thus we took the opportunity to enhance our precalculus course to better prepare the students to learn new material quickly. We also are looking at other solutions such as adding tutorials and raising the threshold score on our calculus placement exam.

We had to examine carefully the impact of the new calculus sequence upon students coming to Trinity with credit by Advanced Placement, International Baccalaureate, or transfer. Early transcendentals did not present a problem, but there may be a tricky transition for students starting in Vector Calculus, as they would not have had any early differential equations. The solution was to insert a brief review into Differential Equations and to monitor the progress of students coming in with course credit.

**Keys to reforming a mathematics curriculum for STEM students**

Keys to carrying out our reform efforts included listening closely to our constituencies' needs, assessing our own departmental needs, building intradepartmental and interdepartmental consensus, designing an ambitious yet flexible curriculum, developing a conscientious assessment plan, and planning to accommodate different pedagogical methods in our reform strategy. However, during our deliberations, many issues arose that are worth examining.

Both departments had to control the urge to turn content reform into the exercise of merely constructing a content "shopping list." We could quickly list all the topics that the engineering students need to know and make sure they are included. But this type of thinking results in the Sisyphean task of preparing students for every possible eventuality, rather than teaching them the fundamentals of mathematical thought and problem solving to instill the confidence and background to tackle a wide variety of quantitative or qualitative problems on their own. In fact we had to remove several conventional topics and traditional exercises to make room for the accelerated delivery. As a result, the content syllabi are briefer, giving latitude to the individual instructors to add topics or applications of their choosing to reinforce the fundamental mathematical training. To our initial surprise, our colleagues in engineering were supportive of abandoning the "shopping list" mentality in favor of developing a more holistic approach. They wanted their students to obtain profound understanding of mathematics as a technical language and as a discipline itself.

Several items were helpful in our curricular revision process. Attending the INTERMATH workshop on as an interdepartmental team really kick started our process.
Obtaining external funding, in the form of small grants from COMAP, Inc., partially supported the interdepartmental retreat, the task forces, and the numerous meetings necessary to achieve our goals. (Attending the INTERMATH workshop alerted us to this funding source.) Writing the grant proposals also gave us to momentum to aim for additional external funding for future curricular reform needs. The retreat allowed the departments to get away together to focus upon the issues in a concentrated time block. Working with our constituent STEM department every step of the way was crucial to solving problems. Finally, obtaining the blessings of other constituent departments who would be affected by the reforms involved them in the process and allowed them to recognize the benefits of curricular reform to their students.

**Less tangible benefits of reform**

This discussion has focused upon the core curriculum we have devised, emphasizing the changes implemented in content delivery. It is also worth mentioning some less tangible benefits of this reform, such as change in both interdepartmental and intradepartmental dynamics. For example, social interactions between the departments improved. Not only did we build some social activities into each of the steps, but our desire to congregate socially, engineers and mathematicians together, continued well after the reform activities concluded.

As a mathematics department, we overcame a big fear of allowing other STEM disciplines a genuine role in setting forth what we teach. Oftentimes, for a variety of reasons, we are territorial about our discipline and reluctant to give up control. But that was a necessary step in our process. Our work with the engineers gave us the courage to look to other constituencies. We now have a plan in progress to develop calculus-based coursework for business and economics students - a program developed by a task force from all three departments. This differs slightly in that our own majors will not be taking this, and it will require additional staffing in order to implement. In this case, the interdepartmental nature of the work has resulted in cultivating strong allies to help us lobby for additional staffing.

What we have done may not seem to have a direct bearing on your own department's issues in core curriculum for STEM students, but perhaps some of the lessons we learned are more universal. Listen to the needs of your constituencies. Involve them in the process from the initial rough ideas to the execution of assessment schemes. Share with them your needs and your plans. Careful consideration of the mathematical training of STEM students will benefit everybody.